Monday, June 27, 2022

* Clarify schedule & exam dates

MATH 164 Lecture Notes

Welcome & Introduction

- Syllabus
  - Exam dates
  - Webwork
  - Participation points
- Schedule overview

Email:

- ewindes@u.rochester.edu
- ewindes@ur.rochester.edu
- ewindes@uoregon.edu

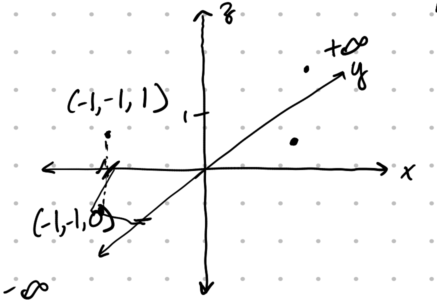
Participation:

1. Come to class (once a week)
2. Come to office hours (once a week)
3. Email answers to questions

Chapter 12: Vectors + the Geometry of Space

Section 12.1: Three-dimensional coordinate systems

- 3d coordinates
- coordinate planes
- octants
- projections
- the right hand rule



xy-plane: $\{(x,y,z) : z=0\}$

xz-plane: $\{(x,y,z) : y=0\}$

yz-plane: $\{(x,y,z) : x=0\}$

$\mathbb{R}^3$ orientation

Octants

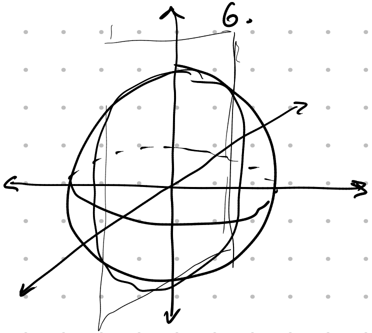
1. $(x,y,z) : x,y,z > 0$
2. $(x,y,z) : x,y > 0,\ z < 0$
3.
4.
5.
6.

□ Surfaces

- Graph of a function of 2-variables:

$$z = \sin\left(\frac{\pi(x^2+y^2)}{2}\right)$$

- Sphere is not the graph of a function.

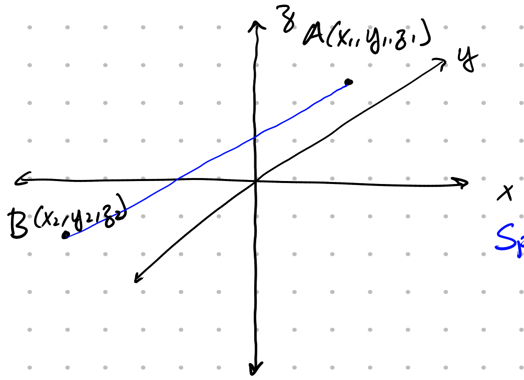


$y = mx + b$

xz-plane: $\{(x,y,z) : y = 0\}$

- Distances and Spheres
  - Distance formula
  - Equation of a sphere

$\{(x,y) : y = mx + b\}$ $A(1,2,3)$



How far is A from B?

$$\boxed{Dist(AB) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

Sphere of radius 1 centered at the origin:

$$S = \{(x,y,z) : dist((0,0,0),(x,y,z)) = 1\}$$

$$1 = x^2 + y^2 + z^2$$

$\Rightarrow$ General equation for a sphere centered at (a,b,c) w/ radius r: $r^2 = (x-a)^2 + (y-b)^2 + (z-c)^2$

Section 12.1 Example Problems

EXAMPLE 6 Show that $x^2 + y^2 + z^2 + 4x - 6y + 2z + 6 = 0$ is the equation of a sphere, and find its center and radius.

$r^2 = (x-a)^2 + (y-b)^2 + (z-c)^2$

$x^2 + 4x + \boxed{4} + y^2 - 6y + \boxed{9} + z^2 + 2z + \boxed{1} = -6 + 4 + 9 + 1$

$\parallel$

$(x+2)^2 + (y-3)^2 + (z+1)^2 = 8$ ✓

radius: $\sqrt{8}$

center: $(-2, 3, -1)$

11. Determine whether the points lie on a straight line.

(a) $A(2, 4, 2)$, $B(3, 7, -2)$, $C(1, 3, 3)$

(b) $D(0, -5, 5)$, $E(1, -2, 4)$, $F(3, 4, 2)$

12. Find the distance from (4, −2, 6) to each of the following.

~~(a)~~ The xy-plane

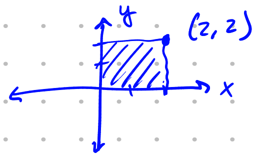
(b) The yz-plane 4

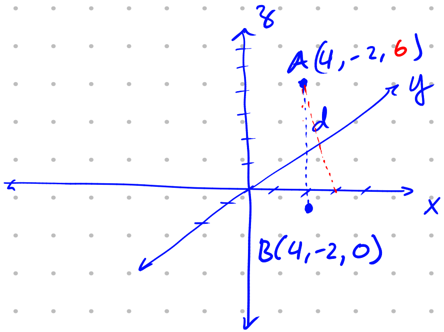
(c) The xz-plane

(d) The x-axis

(e) The y-axis

(f) The z-axis





a):

$$\text{Dist}(AB) = \sqrt{(4-4)^2 + (-2+2)^2 + (6-0)^2}$$

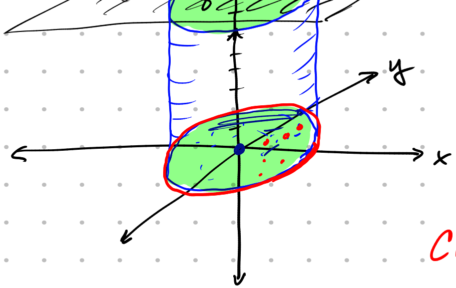
$$= \sqrt{6^2} = 6$$

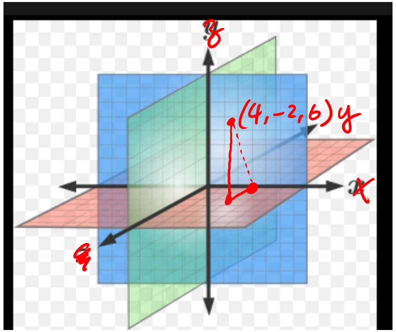
Write inequalities to describe:

40. The solid cylinder that lies on or below the plane $z = 8$ and on or above the disk in the xy-plane with center the origin and radius 2

41. The region consisting of all points between (but not on) the spheres of radius r and R centered at the origin, where $r < R$

42. The solid upper hemisphere of the sphere of radius 2 centered at the origin





$$P_{xy}(4, -2, 6) = (4, -2, 0)$$

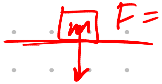
$$P_x(4, -2, 0) = (4, 0, 0)$$

$$\text{Dist}((4, -2, 6), (4, 0, 0))$$

$$= \sqrt{4 + 36} = \sqrt{40}$$

Cylinder: $\{(x, y, z) \in \mathbb{R}^3 : 0 < z \leq 8,\ x^2 + y^2 \leq 4\}$

Section 12.2: Vectors



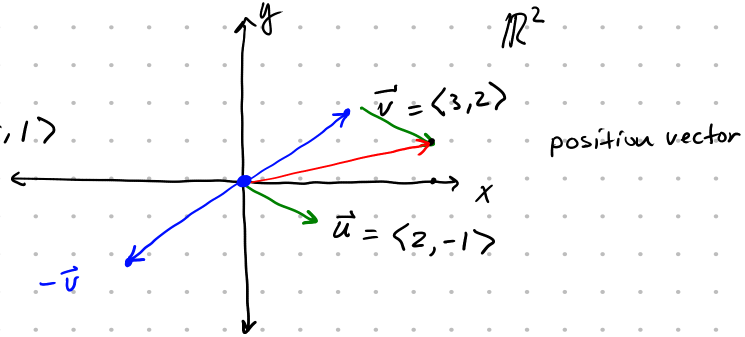
$\vec{F} = m\vec{a}$

- Definition of a vector: something that has length (magnitude) + direction.
- Examples of vectors: velocity of wind, force
- Properties of vectors
  - Addition is associative + commutative

$$\vec{v} + \vec{u} = \langle 3,2\rangle + \langle 2,-1\rangle = \langle 3+2,\ 2-1\rangle = \langle 5,1\rangle$$

$$(\vec{u}+\vec{v}) = (\vec{v}+\vec{u})$$

$$(\vec{u}+\vec{v}) + \vec{w} = \vec{u} + (\vec{v}+\vec{w})$$



  - Existence of zero (only vector with no length or direction)

$$\vec{u} + \vec{0} = \vec{u}$$

  - Existence of identity

$$1 \cdot \vec{a} = \vec{a}$$

  - Existence of inverse $\vec{v} = \langle 3,2\rangle \rightsquigarrow -\vec{v} = \langle -3,-2\rangle$: $\vec{v} + (-\vec{v}) = \vec{0}$

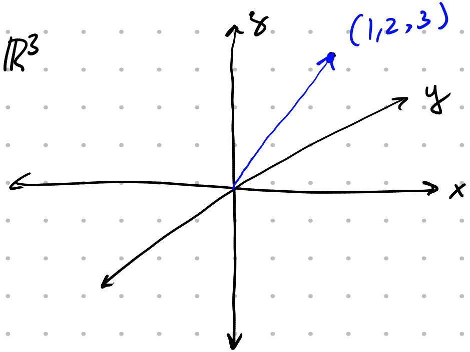
  - scalar multiplication is distributive + associative $\vec{v} = \langle 3,2\rangle$ $\frac{1}{2}\vec{v} = \langle \frac{1}{2}3, \frac{1}{2}2\rangle$

$$a(\vec{v}+\vec{u}) = a\vec{v} + a\vec{u}$$

$$(a+b)\vec{u} = a\vec{u} + b\vec{u} \qquad (ab)\vec{u} = a(b\vec{u})$$

- Standard basis vectors

$\mathbb{R}^3$



Want: 3 vectors in $\mathbb{R}^3$ such that any vector is a "linear combination" of those 3:

$\vec{i}, \vec{j}, \vec{k}$ $\qquad \vec{u} = a\vec{i} + b\vec{j} + c\vec{k}$

Book: uses bold letters for vectors.

$\vec{i} = \langle 1, 0, 0 \rangle$

$\vec{j} = \langle 0, 1, 0 \rangle$

$\vec{k} = \langle 0, 0, 1 \rangle$

$$\vec{u} = \langle 1,2,3 \rangle = 1 \cdot \vec{i} + 2\vec{j} + 3 \cdot \vec{k}$$

- Unit vectors: A vector pointing in the same direction with magnitude $= 1$

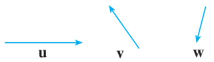
$\vec{u} = \langle a, b, c \rangle$ $\qquad \vec{u} = \langle 1,2,3 \rangle \quad |\vec{u}| = \sqrt{1+4+9} = \sqrt{14}$

$|\vec{u}| = \sqrt{a^2 + b^2 + c^2}$ $\qquad$ Unit vector: $\hat{u} = \frac{1}{|\vec{u}|} \cdot \vec{u} = \left\langle \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right\rangle$

Section 12.2 Example Problems

5. Copy the vectors in the figure and use them to draw the following vectors.

(a) $\mathbf{u} + \mathbf{v}$ (b) $\mathbf{u} + \mathbf{w}$
(c) $\mathbf{v} + \mathbf{w}$ (d) $\mathbf{u} - \mathbf{v}$
(e) $\mathbf{v} + \mathbf{u} + \mathbf{w}$ (f) $\mathbf{u} - \mathbf{w} - \mathbf{v}$



19–22 Find $\mathbf{a} + \mathbf{b}$, $4\mathbf{a} + 2\mathbf{b}$, $|\mathbf{a}|$, and $|\mathbf{a} - \mathbf{b}|$.

19. $\mathbf{a} = \langle -3, 4 \rangle$, $\mathbf{b} = \langle 9, -1 \rangle$

20. $\mathbf{a} = 5\mathbf{i} + 3\mathbf{j}$, $\mathbf{b} = -\mathbf{i} - 2\mathbf{j}$

21. $\mathbf{a} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$, $\mathbf{b} = 2\mathbf{i} - 4\mathbf{k}$

22. $\mathbf{a} = \langle 8, 1, -4 \rangle$, $\mathbf{b} = \langle 5, -2, 1 \rangle$

23–25 Find a unit vector that has the same direction as the given vector.

23. $\langle 6, -2 \rangle$

24. $-5\mathbf{i} + 3\mathbf{j} - \mathbf{k}$

25. $8\mathbf{i} - \mathbf{j} + 4\mathbf{k}$

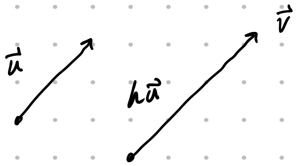
21: $\vec{a} = 4\vec{i} - 3\vec{j} + 2\vec{k}$

$\vec{b} = 2\vec{i} - 4\vec{k}$

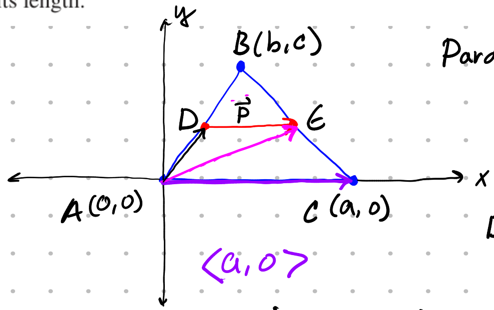
$\vec{a} + \vec{b} = (4+2)\vec{i} - 3\vec{j} + (2-4)\vec{k}$

$= 6\vec{i} - 3\vec{j} - 2\vec{k}$

$= \langle 6, -3, -2 \rangle$



51. Use vectors to prove that the line joining the midpoints of two sides of a triangle is parallel to the third side and half its length.



Parallel: $\vec{u} \parallel \vec{v} \iff \vec{u} = k\vec{v}$

$$\vec{AC} = \langle a, 0 \rangle$$

$D\left(\frac{b}{2}, \frac{c}{2}\right)$ $\quad \left\langle \frac{b}{2}, \frac{c}{2} \right\rangle$ $\quad E\left(\frac{b+a}{2}, \frac{c}{2}\right)$ $\quad \left\langle \frac{b+a}{2}, \frac{c}{2} \right\rangle$

$$\vec{D} + \vec{P} = \vec{E}$$

$$\vec{P} = \vec{E} - \vec{D}$$

$$\vec{DE} = \left\langle \frac{b+a}{2} - \frac{b}{2}, \frac{c}{2} - \frac{c}{2} \right\rangle$$

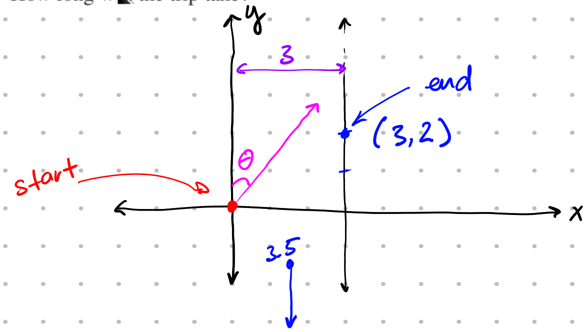
$$= \left\langle \frac{a}{2}, 0 \right\rangle$$

□

39. A boatman wants to cross a canal that is 3 km wide and wants to land at a point 2 km upstream from his starting point. The current in the canal flows at 3.5 km/h and the speed of his boat is 13 km/h.

(a) In what direction should he steer?

(b) How long will the trip take?



$v = d \cdot t$

$\vec{v}_c = \langle 0, -3.5 \rangle$

$\vec{v}_b = \langle 13\cos\theta \, , 13\sin\theta \rangle$

$\left(\langle 13\cos\theta, 13\sin\theta \rangle + \langle 0, -3.5 \rangle \right) = \langle 3, 2 \rangle$

involve time using $v = d \cdot t$

Questions for Participation Points

1. Let $A(-4,0,1)$, $B(3,1,-5)$, and $C(2,4,6)$ be points in $\mathbb{R}^3$.
   Which is closest to the yz-plane? Which lies in the xz-plane?

2. Find a unit vector in the same direction as $\vec{v} = \langle 1,2,3 \rangle$

Right hand rule: