Monday, June 28, 2021
MTH 164 Lecture Notes

Exam Scheduling

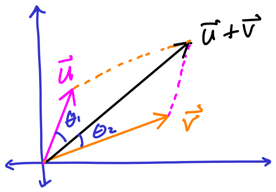
* There will be a new signup sheet
  (You don't have to fill it out but I will use the new one over the old one)

* Final times :

  1. 7am - 10am
  2. 9am - 12pm   Thursday July 1
  3. 6pm - 9pm

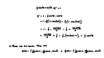
Review Day 1!

Problem 1: If $2\vec{w} = |\vec{u}|\vec{v} + |\vec{v}|\vec{u}$, where $\vec{u}, \vec{v}, \vec{w} \neq 0$ & $\in \mathbb{R}^3$ $(\vec{u} \neq \vec{v})$, show that $\vec{w}$ bisects the angle between $\vec{u}$ & $\vec{v}$.



$$2\vec{w} = |\vec{u}|\cdot|\vec{v}|\cdot\left(\frac{\vec{v}}{|\vec{v}|} + \frac{\vec{u}}{|\vec{u}|}\right) = |\vec{u}||\vec{v}|(\hat{u} + \hat{v})$$

$\Rightarrow \vec{w}$ is a scalar multiple of a vector bisecting $\vec{u}$ & $\vec{v}$

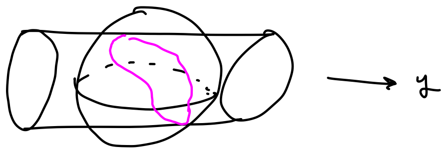


Problem 2: Find parametric equations for the curve of intersection between the sphere $x^2+y^2+z^2=1$ and the cylinder $2x^2+z^2=1$

- $z^2 = 1-2x^2$

$x^2+y^2+1-2x^2 = 1$

$y^2-x^2=0 \Rightarrow \boxed{y^2=x^2}$



- $x = \frac{1}{\sqrt{2}}\cos\theta$

$z = \sin\theta$

$$\boxed{\begin{aligned} C_1(\theta) &= \left\langle \tfrac{1}{\sqrt{2}}\cos\theta, \tfrac{1}{\sqrt{2}}\cos\theta, \sin\theta \right\rangle \\ C_2(\theta) &= \left\langle \tfrac{1}{\sqrt{2}}\cos\theta, \tfrac{-1}{\sqrt{2}}\cos\theta, \sin\theta \right\rangle \end{aligned}}$$

Problem 3: Find the equation of the plane:

$\vec{n}\cdot(\vec{r}-\vec{r}_0) = 0$

$ax+by+cz+d=0$

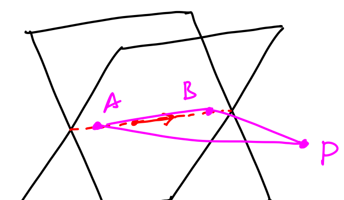
- Through the origin & perpendicular to $\langle 1,-2,5\rangle$
- Through the point $(1,2,3)$ parallel to $\boxed{z=x+y}$ $\langle -1,-1,1\rangle$

$z-x-y=0$ $\quad n = \langle -1,-1,1\rangle$

- Passes through $\boxed{(3,1,4)}$ & contains the line of intersection of $x+2y+3z=1$ and $2x-y+z=-3$

$\langle 2,-1,1\rangle$

$$\begin{vmatrix} & i \; j \; k \\ n_1 = & \langle 1,2,3\rangle \\ n_2 = & \langle 2,-1,1\rangle \end{vmatrix} = \vec{v}$$



$\vec{r}(t) = \vec{r}_0 + t\vec{v}$ $\qquad \langle -1,1,0\rangle = \vec{r}_0$

$\vec{n} = \vec{AB} \times \vec{AP}$

Problem 4: Find the length of the curve $\vec{r}(t) = \langle 2t^{3/2}, \cos(2t), \sin(2t) \rangle$ $\quad 0 \le t \le \frac{\pi}{2}$

$\int_a^b |\vec{r}\,'(t)|\,dt$

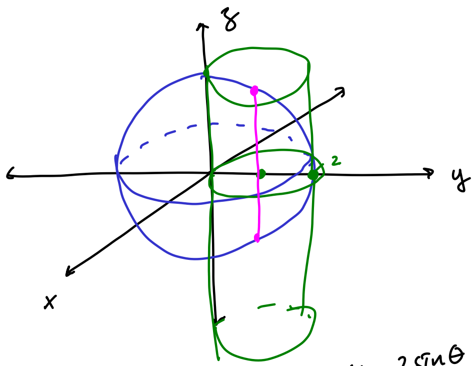
$\vec{r}\,'(t) = \langle 3t^{1/2}, -2\cdot\sin(2t), 2\cos(2t) \rangle$

$|\vec{r}\,'(t)| = \sqrt{9t + 4\sin^2(2t) + 4\cos^2(2t)}$

$= \sqrt{9t + 4}$

$\int_0^{\pi/2} \sqrt{9t+4}\,dt$

Problem 5: Set up the integral representing the volume of the solid contained inside the sphere $4 = x^2 + y^2 + z^2$ & outside the cylinder $x^2 + (y-1)^2 = 1$



$$\boxed{2\int_0^{\pi}\int_{2\sin\theta}^{2}\int_0^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta + 2\int_{\pi}^{2\pi}\int_0^{2}\int_0^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta}$$

$z = \pm\sqrt{4 - x^2 - y^2}$

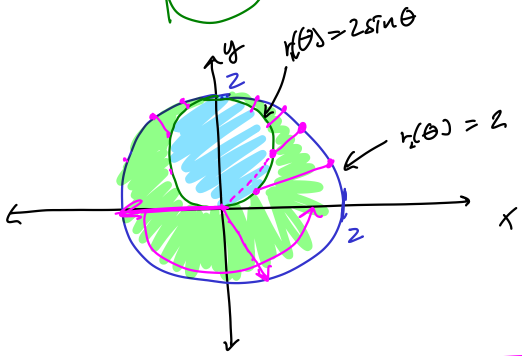
$x = r\cos\theta$

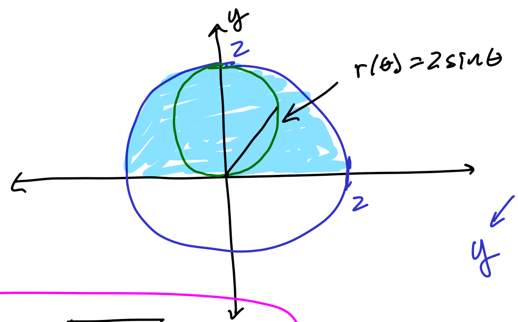
$y = r\sin\theta$

$r^2\cos^2\theta + r^2\sin^2\theta - 2r\sin\theta + 1 = 1$

$r^2 - 2r\sin\theta = 0$

$r^2 = 2\cancel{r}\sin\theta$





yes not understand

y n

$$\boxed{2\int_0^{2\pi}\int_0^{2}\int_0^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta - 2\int_0^{\pi}\int_0^{2\sin\theta}\int_0^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta}$$

Problem 6: $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$ (A) $\qquad \lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$ (B)

- Squeeze
- Polar Coordinates

$r^4 \frac{x^4-y^4}{x^2+y^2}$ $\cancel{r^2}$

A) DNE

when $x=0$,

$$\lim_{(0,y)\to(0,0)} \frac{-y^2}{y^2} = -1$$

when $y=0$

$$\lim_{(x,0)\to(0,0)} \frac{x^2}{x^2} = 1$$

$\therefore$ DNE since $-1 \neq 1$

B) DNE

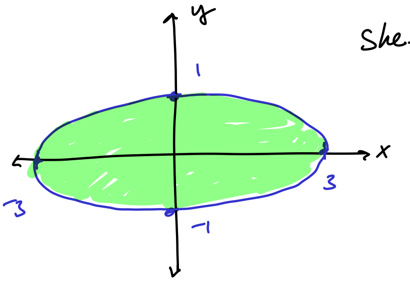
when $x=0$

$$\lim_{(0,y)\to(0,0)} 0 = 0$$

when $x=y^2$

$$\lim_{(y^2,y)\to(0,0)} \frac{y^4}{2y^4} = \frac{1}{2} \neq 0$$

Problem 7: Find & sketch the domain of $\ln(9-x^2-9y^2)$



Sketch $D = \{(x,y): 9-x^2-9y^2 > 0\}$

$\ln(x) = y$

$x = e^y > 0$

$$9-x^2-9y^2 = 0$$
$$9 = x^2+9y^2$$
$$1 = \left(\frac{x}{3}\right)^2 + y^2$$

Problem 8: Find the first-order partial derivatives of $f(x,y) = \int_y^x \cos(e^t)\,dt$

$$\frac{\partial f}{\partial x} = \cos(e^x)$$

$$\frac{\partial f}{\partial y} = -\cos(e^y)$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

Problem 9: What is an equation of the plane tangent to the surface $xy + xz + yz = 4$ at the point $(0,2,2)$?

$$\vec{n}\cdot(\vec{r}-\vec{r_0}) = 0$$

$$F(x,y,z) = xy + xz + yz$$

$$\nabla F = \langle y+z,\ x+z,\ y+x \rangle$$

$$\nabla F(0,2,2) = \langle 4, 2, 2 \rangle$$

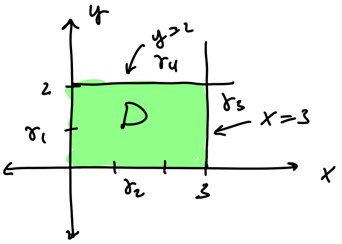
$$\langle 2,1,1 \rangle$$

$$\langle 2,1,1\rangle \cdot \langle x, y-2, z-2 \rangle = 0$$

$$2x + y - 2 + z - 2 = 0$$

$$\boxed{2x + y + z = 4}$$

Problem 10: Find the absolute max & min of $f(x,y) = x^2 - 2xy + 2y$ on the rectangle
$D = \{(x,y) : 0 \le x \le 3,\ 0 \le y \le 2\}$.



1. Check the CP of $f(x,y)$

$f_x(x,y) = 2x - 2y \quad = 0$

$f_y(x,y) = -2x + 2 \quad = 0 \quad \Rightarrow x = 1 \quad \Rightarrow y = 1$

Only cp is $(1,1)$, $f(1,1) = 1 - 2 + 2 = 1$

2. Check sides.

γ_1) when $x = 0$, $f(y) = 2y$ min: $y = 0$ max: $y = 2$

$f(0,0) = 0$ $f(0,2) = 4$

absolute max

γ_2) when $y = 0$ $f(x) = x^2$ min: $x = 0$, max: $x = 3$, $f(3,0) = 9$

γ_3) when $x = 3$ $f(y) = 9 - 6y + 2y = 9 - 4y$ $\quad x^2 - 2xy + 2y$

min: $y = 2$ $f(3,2) = 1$

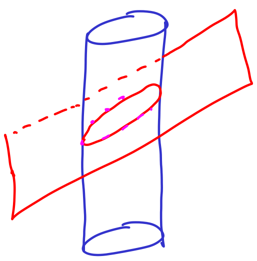
γ_4) when $y = 2$ $f(x) = x^2 - 4x + 4$

$f'(x) = 2x - 4 = 0 \Rightarrow x = 2$

absolute min

$f(2,2) = 4 - 8 + 4 = 0$

Problem 11: Find the max value of $f(x,y,z) = x + 2y + 3z$ on the intersection of the plane $\underline{x - y + z = 1}$ & the cylinder $\underline{x^2 + y^2 = 1}$



$g(x,y,z) = x - y + z$

$h(x,y,z) = x^2 + y^2$

$\nabla f = \lambda \nabla g + \mu \nabla h$

$\nabla f = \langle 1, 2, 3 \rangle$

$\nabla g = \langle 1, -1, 1 \rangle$

$\nabla h = \langle 2x, 2y, 0 \rangle$

$1 = 3 + 2\mu x \rightsquigarrow -2 = 2\mu x \rightsquigarrow -1 = \mu x \rightsquigarrow x = \frac{-1}{\mu}$

$2 = -3 + 2\mu y \rightsquigarrow 5 = 2\mu y \rightsquigarrow y = \frac{5}{2\mu}$

$x - y + z = 1$

$x^2 + y^2 = 1$

$\frac{1}{\mu^2} + \frac{25}{4\mu^2} = 1 \Rightarrow \mu^2 = 1 + \frac{25}{4}$

Problem 12: Find the positively oriented, simple, closed curve C that maximizes the integral

$$\int_C \underbrace{(y^3-y)}_{P}\,dx - \underbrace{2x^3}_{Q}\,dy$$

$$\frac{\partial Q}{\partial x} = -6x^2 \qquad \frac{\partial P}{\partial y} = 3y^2 - 1$$

$$\int_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

Green's theorem.

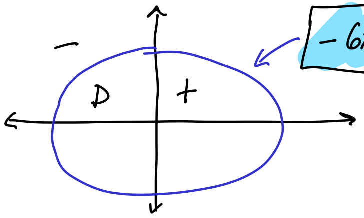
The answer is the boundary of the domain $D = \{(x,y) : f(x,y) \geq 0\}$

$$\int_C (y^3-y)\,dx - 2x^3\,dy = \iint_D \underbrace{\left(-6x^2 - 3y^2 + 1\right)}_{f(x,y)} dA$$

when is $f(x,y) > 0$?

$$\boxed{f(x,y) = -6x^2 - 3y^2 + 1}$$

max of $f(x,y) = 1$ & happens when $(x,y) = (0,0)$

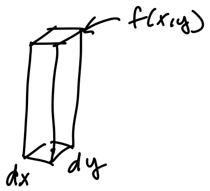


$$\boxed{-6x^2 - 3y^2 + 1 = 0}$$

$$1 = 6x^2 + 3y^2$$

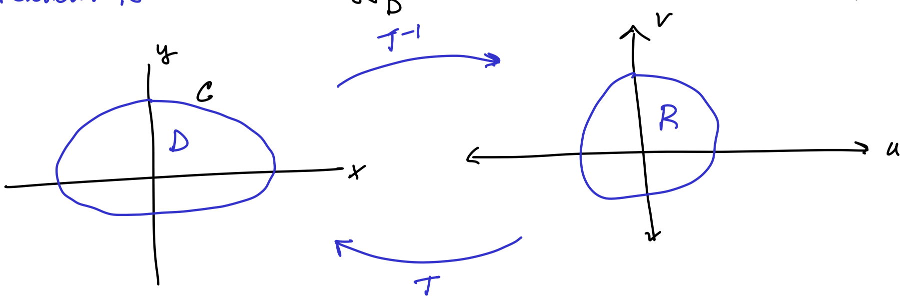
$$\frac{6\cos^2 t}{6} + \frac{3\sin^2}{3} = 1$$

$$\therefore\ c(t) = \left\langle \frac{\cos t}{\sqrt{6}}, \frac{\sin t}{\sqrt{3}} \right\rangle$$



$$\sum_{i=1}^{n} f(x_i^*, y_i^*)\,\Delta x_i\,\Delta y_i$$

Problem 13: Evaluate $\iint_D dA$ where D is bounded by $6x^2 + 3y^2 = 1$



$$x = \frac{u}{\sqrt{6}}$$

$$y = \frac{v}{\sqrt{3}}$$

$$T(u,v) = \left(\frac{u}{\sqrt{6}}, \frac{v}{\sqrt{3}}\right)$$

$$J(T) = \begin{vmatrix} \frac{1}{\sqrt{6}} & 0 \\ 0 & \frac{1}{\sqrt{3}} \end{vmatrix} = \frac{1}{\sqrt{18}}$$

$$\iint_D dA_D = \iint_R \frac{1}{\sqrt{18}}\,dA_R = \frac{\pi}{\sqrt{18}}$$

$$\boxed{\pi \cdot a \cdot b}$$