

MATH 201: LINEAR ALGEBRA
SUGGESTED PROBLEMS FOR WEEK 2

1. BASIC SKILLS

Problem 1.1. Consider a linear system of three equations with three unknowns. We are told that the system has a unique solution. What does the reduced row-echelon form of the coefficient matrix of this system look like?

Problem 1.2. Consider a linear system of four equations with three unknowns. We are told that the system has a unique solution. What does the reduced row-echelon form of the coefficient matrix of this system look like?

Problem 1.3. Let

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}, \quad B = \begin{bmatrix} 12 & 11 & 10 & 9 \\ 8 & 7 & 6 & 5 \\ 4 & 3 & 2 & 1 \end{bmatrix}.$$

Find $A + B$ and $2A + 3B$.

Problem 1.4. Let

$$\vec{u} = \begin{bmatrix} 1 \\ 3 \\ -5 \\ 2 \\ 4 \end{bmatrix} \quad \text{and} \quad \vec{v} = \begin{bmatrix} 2 \\ -1 \\ 4 \\ -3 \\ 5 \end{bmatrix}.$$

Find the dot product $\vec{u} \cdot \vec{v}$.

Problem 1.5. Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \quad \text{and} \quad \vec{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Compute the product $A\vec{x}$.

Problem 1.6. Given the vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$, determine if the vector

$$\vec{y} = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$

can be expressed as a linear combination of \vec{v}_1 and \vec{v}_2 .

Problem 1.7. Consider the following system of linear equations:

$$\begin{cases} 2x_1 + 3x_2 - x_3 = 7 \\ 4x_1 - 2x_2 + 5x_3 = 8 \\ -3x_1 + x_2 + 2x_3 = -4 \end{cases}$$

Find the matrix form of this system of equations, that is, express it in the form $A\vec{x} = \vec{b}$, where A is the coefficient matrix, \vec{x} is the vector of variables, and \vec{b} is a constant vector.

2. TYPICAL PROBLEMS

Problem 2.1. If the rank of a 4×4 matrix A is 4, what is $\text{rref}(A)$? If the rank of a 5×3 matrix B is 3, what is $\text{rref}(B)$?

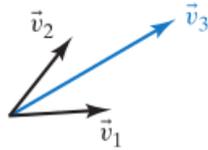
Problem 2.2. True or false?

- The linear system $A\vec{x} = \vec{b}$ is consistent if (and only if) $\text{rank}(A) = \text{rank}[A|\vec{b}]$.
- If A and B are matrices of the same size, then the formula $\text{rank}(A + B) = \text{rank}(A) + \text{rank}(B)$ must hold.
- The rank of any upper triangular matrix is the number of nonzero entries on its diagonal.
- If $A = [\vec{u}, \vec{v}, \vec{w}]$ and $\text{rref}(A) = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix}$, then the equation $\vec{w} = 2\vec{u} + 3\vec{v}$ must hold.
- If A is any 4×3 matrix, then there exists a vector \vec{b} in \mathbb{R}^4 such that the system $A\vec{x} = \vec{b}$ is inconsistent.

Problem 2.3. Consider the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ in \mathbb{R}^2 shown in the picture below. How many solutions x, y does the system

$$x\vec{v}_1 + y\vec{v}_2 = \vec{v}_3$$

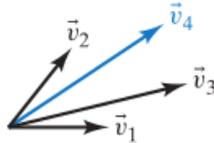
have?



Problem 2.4. Consider the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4$ in \mathbb{R}^2 shown in the picture below. Find **two** solutions x, y, z of the linear system

$$x\vec{v}_1 + y\vec{v}_2 + z\vec{v}_3 = \vec{v}_4.$$

How do you know this system has, in fact, infinitely many solutions?



3. CHALLENGE PROBLEMS

Problem 3.1. Consider the matrix

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & t \\ 1 & 1 & 1 \end{pmatrix}.$$

a) Compute $\text{rank}(A(t))$ as a function of t . For which values of t is the solution to the equation $A(t)\vec{x} = \vec{b}$ unique for every \vec{b} ?

b) At the value(s) of t where $\text{rank}(A(t)) < 3$, characterize all \vec{b} for which $A(t)\vec{x} = \vec{b}$ is solvable. When solvable, how many solutions are there?

The problem below comes from Section 1.8 of Ken Kuttler's Linear Algebra book on *Libre-Texts*. It requires Kirchoff's Law:

Theorem (Kirchoff's Law). *The sum of the resistance (R) times the amps (I) in the counter clockwise direction around a loop equals the sum of the voltage sources (V) in the same direction aroundn the loop.*

Problem 3.2. The diagram below consists of four circuits. The current (I_k) in the four circuits is denoted by I_1, I_2, I_3, I_4 . Using Kirchoff's Law, write an equation for each circuit and solve for each current.

