MATH 201: LINEAR ALGEBRA
SUGGESTED PROBLEMS FOR WEEK 1

1. Basic Skills

Problem 1.1. Find *all* solutions to the systems. Represent your solutions geometrically.

$$\begin{cases} x - 2y = 2 \\ 3x + 5y = 17 \end{cases} \qquad \begin{cases} x - 2y = 3 \\ 2x - 4y = 6 \end{cases} \qquad \begin{cases} x - 2y = 2 \\ 2x - 4y = 8 \end{cases}$$

Problem 1.2. Find the augmented and coefficient matrices of the linear system of equations.

$$\begin{cases} x_1 = -3 \\ -3x_1 + x_2 = 14 \\ x_1 + 2x_2 + x_3 = 9 \\ -x_1 + 8x_2 - 5x_3 + x_4 = 33 \end{cases}$$

Problem 1.3. Find the augmented and coefficient matrices of the linear system of equations.

$$\begin{cases} x_7 = x_1 + x_5 \\ x_1 + x_2 = x_8 \\ x_3 = x_2 + x_4 \\ x_4 + x_5 = x_9 + x_6 \end{cases}$$

Problem 1.4. Which of the matrices are in rref? For those that are *not* in rref, say which condition is violated.

$$\begin{bmatrix} 1 & 2 & 0 & 2 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \end{bmatrix}$$

Problem 1.5. Solve the system of equations using Gauss–Jordan elimination.

$$\begin{cases} 3x + 5y + 3z = 25 \\ 7x + 9y + 19z = 65 \\ -4x + 5y + 11z = 5 \end{cases}$$

2. Typical Problems

Problem 2.1. We say that two $n \times m$ matrices in rref are of the same *type* if they contain the same number of leading 1's in the same positions. How many types of 3×2 matrices in rref are there?

Problem 2.2. Is there a sequence of elementary row operations that transforms

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \text{into} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}?$$

Problem 2.3. For which values of $a, b, c, d,$ and e is the following matrix in rref?

$$\begin{bmatrix} 1 & a & b & 3 & 0 & -1 \\ 0 & 0 & c & 1 & d & 3 \\ 0 & e & 0 & 0 & 1 & 1 \end{bmatrix}$$

3. Challenge Problems

Problem 3.1. Queens, New York has several one-way streets throughout its many neighborhoods. We can represent the flow of the traffic around 81st and 82nd streets diagrammatically as

Imagine that we send out detectors (such as scouts) to record the average number of cars per hour along each street. What is the smallest number of scouts we will need to determine the traffic flow on every street?

Problem 3.2. Consider the chemical reaction

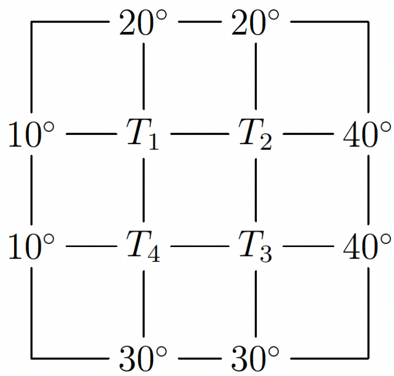
$$aNO_2 + bH_2O \longrightarrow cHNO_2 + dHNO_3,$$

where a, b, c, and d are unknown positive integers. The reaction must be balanced; that is, the number of atoms of each element must be the same before and after the reaction. For example, because the number of oxygen atoms must remain the same,

$$2a + b = 2c + 3d.$$

Find the smallest possible positive integers a, b, c, and d that balance the reaction.

Problem 3.3. The temperature on the boundary of a cross section of a metal beam is fixed and known but is unknown at the intermediate points on the interior



Assume the temperature at these intermediate points equals the average of the temperature at the nearest neighboring points. Calculate the temperatures T_1, T_2, T_3, T_4.