

Tutorial 2

1. Consider a linear system of three equations with three unknowns. We are told that the system has a unique solution. What does the reduced row-echelon form of the coefficient matrix of this system look like?

2. Consider a linear system of four equations with three unknowns. We are told that the system has a unique solution. What does the reduced row-echelon form of the coefficient matrix of this system look like?

3. Find the sum of the matrices A and B :

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}, \quad B = \begin{bmatrix} 12 & 11 & 10 & 9 \\ 8 & 7 & 6 & 5 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

4. Find $2A + 3B$ for the A and B defined above.

5. Let $\mathbf{u} = \begin{bmatrix} 1 \\ 3 \\ -5 \\ 2 \\ 4 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 2 \\ -1 \\ 4 \\ -3 \\ 5 \end{bmatrix}$. Find the dot product of these vectors.

6. Let A be a 3×2 matrix and \mathbf{x} be a column vector of size 2×1 :

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \quad \text{and} \quad \mathbf{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Compute the product $A\mathbf{x}$.

7. Given the vectors: $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$, determine if the vector

$$\mathbf{y} = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$

can be expressed as a linear combination of \mathbf{v}_1 and \mathbf{v}_2 .

8. Let A be a 2×3 matrix and \mathbf{x} and \mathbf{y} be vectors in \mathbb{R}^3 defined as follows:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Verify the algebraic rule:

$$A(\mathbf{x} + \mathbf{y}) = A\mathbf{x} + A\mathbf{y}$$

9. Let A be a 3×2 matrix and \mathbf{x} be a vector in \mathbb{R}^2 defined as follows:

$$A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \\ 4 & 2 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

Verify the algebraic rule:

$$A(k\mathbf{x}) = k(A\mathbf{x})$$

for $k = 3$.

10. Consider the following system of linear equations:

$$\begin{cases} 2x_1 + 3x_2 - x_3 = 7 \\ 4x_1 - 2x_2 + 5x_3 = 8 \\ -3x_1 + x_2 + 2x_3 = -4 \end{cases}$$

Find the matrix form of this system of equations, i.e., express it in the form $A\mathbf{x} = \mathbf{b}$, where A is the coefficient matrix, \mathbf{x} is the vector of variables, and \mathbf{b} is the constant vector.