Tutorial 1

1. Find the augmented matrix of the linear system of equations

$$\begin{cases} x_1 = -3 \\ -3x_1 + x_2 = 14 \\ x_1 + 2x_2 + x_3 = 9 \\ -x_1 + 8x_2 - 5x_3 + x_4 = 33 \end{cases}$$

2. Solve the system of equations in 1 using Gauss-Jordan elimination.

3. Find the augmented matrix of the linear system of equations

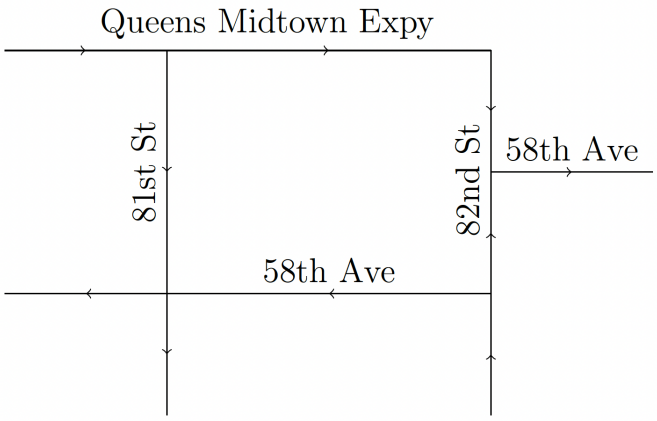
$$\begin{cases} x_7 = x_1 + x_5 \\ x_1 + x_2 = x_8 \\ x_3 = x_2 + x_4 \\ x_4 + x_5 = x_9 + x_6 \end{cases}$$

4. Boris and Marina are shopping for chocolate bars. Boris observes, "If I add half my money to yours, it will be enough to buy two chocolate bars." Marina naively asks, "If I add half my money to yours, how many can we buy?" Boris replies, "One chocolate bar." How much money did Boris have? (From Yuri Chernyak and Robert Rose, The Chicken from Minsk, Basic Books, 1995.

5. For which values of a, b, c, d, and e is the following matrix in reduced row-echelon form?

$$\begin{bmatrix} 1 & a & b & 3 & 0 & -2 \\ 0 & 0 & c & 1 & d & 3 \\ 0 & e & 0 & 0 & 1 & 1 \end{bmatrix}$$

6. Queens, New York has several one-way streets throughout its many neighborhoods. We can represent the flow of the traffic around 81st and 82nd streets diagrammatically as



Imagine that we send out detectors (such as scouts) to record the average number of cars per hour along each street. What is the smallest number of scouts we will need to determine the traffic flow on every street?

Hint: The net flow into an intersection equals the net flow out of an intersection. Each edge connecting any two intersections represents an unknown and each fact above provides an equation. Hence, this system has 9 unknowns and 4 equations.

7. Compute the rank of the following matrices:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}.$$

Reminder: The rank of a matrix is defined as the number of leading (pivot) 1s in the reduced row echelon form of the matrix.