MATH 201: Linear Algebra – Week 11 Quiz

NAME: ____________________________ ID Number: ________________

Problem 1 (3 points)**.** Let P_3 denote the set of all polynomials in one variable of degree at most 3. That is,

$$P_3 = \left\{p(x) \in C^\infty \mid p(x) = a + bx + cx^2 + dx^3, \text{ where } a, b, c, d \in \mathbb{R}\right\}.$$

What is the *dimension* of P_3? Write a basis for P_3.

There is an isomorphism $P_3 \to \mathbb{R}^4$ given by $a + bx + cx^2 + dx^3 \mapsto \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$

Thus the dimension of P_3 is 4.

A basis for P_3, corresponding to the standard basis under the isomorphism, is

$$\{1, x, x^2, x^3\}$$

Problem 2 (3 points)**.** Let $V = \text{span}\{x, y\} \subset C^\infty$. Note that by definition, $\mathcal{B} = \{x, y\}$ is a *basis* for V. Define $T : V \to V$ by

$$T(p(x, y)) = x\frac{d}{dy}p(x, y).$$

Write a matrix for T with respect to the basis $\mathcal{B}$. Is T an *isomorphism*?

Hint: A linear transformation is called an isomorphism if it is invertible.

Note that $V = \{p(x, y) = ax + by\} \cong \mathbb{R}^2$

$$\frac{d}{dy}(ax + by) = b \Rightarrow T(p(x, y)) = bx$$

Under the isomorphism with $\mathbb{R}^2$ we have

$$T(\begin{bmatrix} a \\ b \end{bmatrix}) = \begin{bmatrix} b \\ 0 \end{bmatrix}$$

Thus the matrix representing T in this basis is $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

Clearly, A is not invertible so T is not an isomorphism.

Problem 3 (4 points)**.** Let $W = \text{span}\{\vec{u}_1, \vec{u}_2\}$ where

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

and

$$\vec{u}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}.$$

Find the *orthogonal projection* of $\vec{v} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$ onto W.

Hint: Recall that if $\{u_1, \cdots, u_k\}$ is an *orthogonal basis* for a subspace W of $\mathbb{R}^n$, then

$$\text{proj}_W(\vec{v}) = \sum_{i=1}^{k} \frac{\vec{v} \cdot \vec{u}_i}{\vec{u}_i \cdot \vec{u}_i} \vec{u}_i.$$

$\vec{v} \cdot \vec{u}_1 = 3 + 1 = 4$

$\vec{v} \cdot \vec{u}_2 = 3 - 1 = 2$

$\vec{u}_1 \cdot \vec{u}_1 = 2$

$\vec{u}_2 \cdot \vec{u}_2 = 1 + 1 = 2$

thus $\text{proj}_W(\vec{v}) = \frac{4}{2} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \frac{2}{2} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$

Note that this makes sense because the vectors $\vec{u}_1$ and $\vec{u}_2$ span the xy-plane.

Problem 4 (0 points)**.** Let $z = 2 + 3i$ and $w = 1 - i$. Find the real and imaginary parts of zw. If you do not know how to do this computation, that is okay. In that case, please circle one:

- I have never seen complex numbers before.
- I have seen complex numbers before but do not remember how to multiply them.

$$zw = (2 + 3i)(1 - i) = 2 - 2i + 3i - 3i^2 = 2 + i + 3 = 5 + i$$

where we have used the fact that $i^2 = -1$

Thus $\Re(zw) = 5$ and $\Im(zw) = 1$