MATH 201: Linear Algebra – Week 5 Quiz

NAME: ______________________________ ID Number: __________________

Problem 1 (5 points)**.** Circle the matrices which are invertible.

$$A_1 = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \quad A_5 = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix},$$

Problem 2 (2 points)**.** Suppose that $T : \mathbb{R}^n \to \mathbb{R}^n$ is an invertible, linear transformation. What is the *definition* of its inverse $T^{-1} : \mathbb{R}^n \to \mathbb{R}^n$?

The inverse is the unique function $T^{-1} : \mathbb{R}^n \to \mathbb{R}^n$ such that

$$T^{-1}(T(x)) = T(T^{-1}(x)) = x \qquad \forall x \in \mathbb{R}^n.$$

Problem 3 (3 points)**.** Consider the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$.

$$T : \begin{bmatrix} x \\ y \\ z \end{bmatrix} \mapsto \begin{bmatrix} x+y \\ y+z \\ x+y+z \end{bmatrix}.$$

What is the **kernel** of T? Is T invertible?

The matrix representing T is

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

This has rank 3 so T is invertible.