

Solutions

NAME: _____

ID Number: _____

Problem 1 (3 points). What is a linear transformation?

A function $T : \mathbb{R}^m \rightarrow \mathbb{R}^n$ is called a linear transformation if there exists an $n \times m$ matrix A such that

$$T(\vec{x}) = A\vec{x}$$

for all $\vec{x} \in \mathbb{R}^m$.

Problem 2 (3 points). Consider the following function $T : \mathbb{R}^4 \rightarrow \mathbb{R}^2$. Find a matrix A such that $T(\vec{x}) = A\vec{x}$.

$$T : \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \mapsto \begin{bmatrix} x_1 + x_3 \\ x_2 + x_4 \end{bmatrix}.$$

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Problem 3 (4 points). Compute:

$$(a) \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}$$

$$(b) \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} = \text{NOT defined! } 2 \text{ does not equal } 3$$

$2 \times 2 \quad 3 \times 2$