Solutions

MATH 201: Linear Algebra – Week 3 Quiz

NAME: ______________________ ID Number: ______________

Problem 1 (3 points). Each of the matrices below represents the reduced row echelon form of the *augmented matrix* of a system of linear equations. Say whether the system has **no solution, a unique solution**, or **infinitely many solutions**.

(a) $\left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 0 & 1 \end{array}\right]$. no solutions

(b) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$. unique solution

(c) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \end{array}\right]$ infinitely many solutions

Problem 2 (4 points). For each of the matrices below, determine its **rank**. (The matrices are already in reduced row echelon form).

(a) $\begin{bmatrix} 0 & 1 & -4 & 0 & 2 \\ 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ 2

(b) $\begin{bmatrix} 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ 2

(c) $\begin{bmatrix} 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$ 2

(d) $\begin{bmatrix} 1 & 0 & 2 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ 3

Problem 3 (2 points). Define the term **linear combination**. That is, what does it mean for a vector $\vec{b} \in \mathbb{R}^n$ to be a *linear combination* of the vectors $\vec{v}_1, \dots, \vec{v}_k \in \mathbb{R}^n$?

It means that there exists constants (scalars) $x_1, \dots, x_k \in \mathbb{R}$ such that

$$\vec{b} = x_1\vec{v}_1 + \cdots + x_k\vec{v}_k$$

Problem 4 (1 point). Find a matrix A and vectors $\vec{x}$, and $\vec{b}$ such that the equation $A\vec{x} = \vec{b}$ represents the system below

$$\begin{aligned} x + 2y &= 0 \\ x - y &= 1. \end{aligned}$$

You do *not* have to solve the system.

$$A = \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix} \quad \vec{x} = \begin{bmatrix} x \\ y \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$