

MATH 201 Linear Algebra – Quiz 1A

Fall, 2025

Solutions

Problem 1. Consider the matrices given below. Circle the matrices which are in **reduced row-echelon form**.

$$A = \begin{pmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$D = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$E = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Problem 2. Consider the following system of linear equations.

$$\begin{aligned} 2x_1 + 3x_2 &= 5 \\ x_2 + 4x_3 - x_4 &= 0 \\ x_1 - x_2 &= 0. \end{aligned}$$

a) Write the augmented and coefficient matrices for the system.

$$\begin{bmatrix} 2 & 3 & 0 & 0 \\ 0 & 1 & 4 & -1 \\ 1 & -1 & 0 & 0 \end{bmatrix}$$

coefficient matrix

$$\left[\begin{array}{cccc|c} 2 & 3 & 0 & 0 & 5 \\ 0 & 1 & 4 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 \end{array} \right]$$

augmented matrix

b) Put the augmented matrix from part a) in RREF. Which variables, if any, are free?

$$\begin{aligned} &\left[\begin{array}{cccc|c} 2 & 3 & 0 & 0 & 5 \\ 0 & 1 & 4 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 \end{array} \right] && \begin{array}{l} R_1 \leftrightarrow R_3 \\ -2R_1 + R_3 \rightarrow R_3 \end{array} && \left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 4 & -1 & 0 \\ 2 & 3 & 0 & 0 & 5 \end{array} \right] \\ &&&&&&&&&&&& \left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 4 & -1 & 0 \\ 0 & 5 & 0 & 0 & 5 \end{array} \right] && \begin{array}{l} -5R_2 + R_3 \rightarrow R_3 \\ -\frac{1}{20}R_3 \rightarrow R_3 \end{array} && \left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 4 & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{1}{4} \end{array} \right] \\ &&&&&&&&&&&& \left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{1}{4} \end{array} \right] && \begin{array}{l} -4R_3 + R_2 \rightarrow R_2 \\ R_1 + R_2 \rightarrow R_1 \end{array} && \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{1}{4} \end{array} \right] \\ &&&&&&&&&&&& \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{1}{4} \end{array} \right] && \begin{array}{l} \text{X}_4 \text{ is free} \end{array} && \end{aligned}$$

c) How many solutions does the system have?

infinitely many (1-dimensional space).

Problem 3. Write a matrix representing the **augmented matrix** of a system of 3 equations in 4 variables which has **no solutions**.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right]$$

Problem 4. Let $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 2 & 1 \\ 0 & 0 & -1 \end{pmatrix}$. What is $A + 3B$?

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 6 & 3 \\ 0 & 0 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 8 & 3 \\ 0 & 1 & -3 \end{bmatrix}$$

Problem 5. Let $M = \begin{pmatrix} 1 & 0 \\ 0 & 3 \\ 2 & 1 \end{pmatrix}$. Let $\vec{u} = (4 \ 1 \ 3)$ and $\vec{v} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$.

a) What is $M\vec{u}$?

Not defined

b) What is $M\vec{v}$?

$$\begin{bmatrix} 1 & 0 \\ 0 & 3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 8+1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 17 \end{bmatrix}$$