MATH 201: Linear Algebra – Quiz 3

NAME: ____________________________ ID Number: _________________

Problem 1. Let $\vec{x} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ and $\mathcal{B} = \left\{ \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$. Write $[\vec{x}]_{\mathcal{B}}$.

Problem 2. Suppose that $\mathcal{B} = \left\{ \vec{b}_1, \vec{b}_2 \right\}$ and $\mathcal{B}' = \left\{ \vec{b}'_1, \vec{b}'_2 \right\}$ are two bases for $\mathbb{R}^2$. Write a matrix A (in terms of $\vec{b}_i$, and $\vec{b}'_i$) such that

$$A\,[\vec{x}]_{\mathcal{B}} = [\vec{x}]_{\mathcal{B}'} \qquad \forall\,\vec{x} \in \mathbb{R}^2.$$

Problem 3. Show that
$$W = \left\{ \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \mid a, b \in \mathbb{R} \right\}$$
is a *subspace* of the *linear space* $V = \mathbb{R}^{2\times 2}$.

Problem 4. On $\mathbb{R}$ define exotic addition $\oplus$ and multiplication $\odot$ operations by
$$x \oplus y = x + y + 3 \qquad a \odot x = ax + 3a - 3.$$
It turns out that $V = (\mathbb{R}, \oplus, \odot)$ is a linear space.

(a) Identify the *additive identity* of V.

(b) Identify the *additive inverse* of V.

(c) *Verify* that $a \odot (x \oplus y) = (a \odot x) \oplus (a \odot y)$.