MATH 201 Linear Algebra

Fall, 2025

Problem Set 1

Problem 1. Solve the system of equations: $\begin{cases} 3x + 5y + 3z &= 25 \\ 7x + 9y + 19z &= 65 \\ -4x + 5y + 11z &= 5 \end{cases}$

Problem 2. Is there a sequence of elementary row operations that transforms

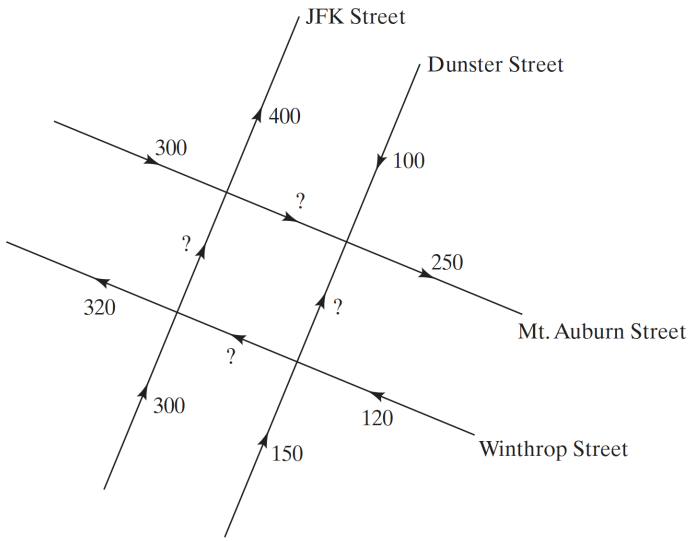
$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \text{ into } \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}?$$

Explain.

Problem 3. Solve the system of equation: $\begin{cases} 3x_1 + 6x_2 + 9x_3 + 5x_4 + 25x_5 &= 53 \\ 7x_1 + 14x_2 + 21x_3 + 9x_4 + 53x_5 &= 105 \\ -4x_1 - 8x_2 - 12x_3 + 5x_4 - 10x_5 &= 11 \end{cases}$

Problem 4. Find the rank of the augmentation matrix of the system of equations in the previous problem.

Problem 5. The accompanying sketch represents a maze of oneway streets in a city in the United States. The traffic volume through certain blocks during an hour has been measured. Suppose that the vehicles leaving the area during this hour were exactly the same as those entering it.



What can you say about the traffic volume at the four locations indicated by a question mark? Can you figure out exactly how much traffic there was on each block? If not, describe one possible scenario. For each of the four locations, find the highest and the lowest possible traffic volume.

Problem 6. Consider the chemical reaction

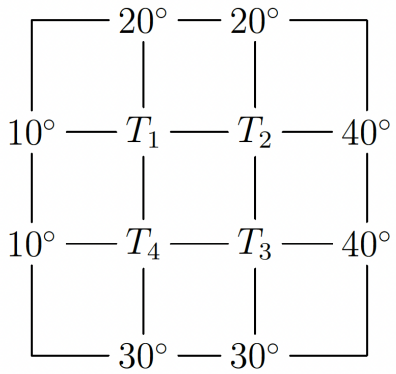
$$aNO_2 + bH_2O \longrightarrow cHNO_2 + dHNO_3,$$

where a, b, c, and d are unknown positive integers. The reaction must be balanced; that is, the number of atoms of each element must be the same before and after the reaction. For example, because the number of oxygen atoms must remain the same,

$$2a + b = 2c + 3d.$$

Find the smallest possible positive integers a, b, c, and d that balance the reaction.

Problem 7. The temperature on the boundary of a cross section of a metal beam is fixed and known but is unknown at the intermediate points on the interior



Assume the temperature at these intermediate points equals the average of the temperature at the nearest neighboring points. Calculate the temperatures T_1, T_2, T_3, T_4.