Math 201: Linear Algebra

Vocabulary Review

November 3, 2025

True or False #1

If A is $n \times m$ with $\operatorname{rank}(A) = m$, then the only solution to $A\mathbf{x} = \mathbf{0}$ is $\mathbf{x} = \mathbf{0}$.

Show Answer

Answer #1

True. Nullity $= m - \text{rank}(A) = 0$, so $\ker(A) = \{\mathbf{0}\}$.

Next Question

True or False #2

If a linear system is consistent, then it has either exactly one solution or infinitely many solutions.

Show Answer

Answer #2

True. Linear systems cannot have exactly k solutions unless $k = 0$ or 1.

Next Question

True or False #3

A linear system with fewer equations than unknowns must have infinitely many solutions.

Show Answer

Answer #3

False. It could be inconsistent; It has "infinitely many" solutions only if it is consistent.

Next Question

True or False #4

The matrix $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is in rref.

Show Answer

Answer #4

True. A matrix is in rref if. . .

1. The first nonzero entry of each row is a 1. These are called the *leading 1s* or *pivots*.
2. Each leading 1 is the *only* nonzero entry in *its column*.
3. Each leading 1 is to the right of any leading ones in the rows above it.
4. If a row of all zeros is present, it is the last row.

Next Question

True or False #5

A system of equations $A\vec{x} = \vec{b}$ is *consistent* if and only if rref $\left[A|\vec{b}\right]$ is equal to the identity matrix.

Show Answer

Answer #5

False. Example: The matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, interpreted as an augmented matrix, says that $x = 0$ and $0 = 1$ which is clearly a contradiction.

Next Question

True or False #6

If B is $n \times p$ and A is $q \times m$, then BA is defined whenever $n = m$.

Show Answer

Answer #6

False. BA is defined iff the inner sizes match: $p = q$, not $n = m$.

Next Question

True or False #7

If a matrix A commutes with a matrix B and if B commutes with the matrix C, then A must commute with C.

Show Answer

Answer #7

False. Counterexample: $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

Next Question

True or False #8

For any $n \times m$ matrix A, $AI_m = I_n A = A$.

Show Answer

Answer #8

True. Identity matrices act neutrally on compatible sides.

Next Question

True or False #9

The image of a linear transformation is always a subspace of the codomain.

Show Answer

Answer #9

True. Closed under addition and scalar multiplication, and contains $\mathbf{0}$.

Next Question

True or False #10

The kernel of a linear transformation is always a subspace of the domain.

Show Answer

Answer #10

True. Same closure properties as above.

Next Question

True or False #11

A set of m vectors in an m-dimensional subspace is called a **basis** for that subspace.

Show Answer

Answer #11

False. They must also be linearly independent. That is, their span must equal the subspace.

Next Question

True or False #12

The vectors in $\ker(A)$ encode exactly the linear relations among the columns of A.

Show Answer

Answer #12

True. $A\mathbf{x} = \mathbf{0}$ expresses column relations with coefficients $\mathbf{x}$.

Next Question

True or False #13

If A is invertible, then $\mathrm{rref}(A) = I_n$ and $\mathrm{rank}(A) = n$.

Show Answer

Answer #13

True. These are standard equivalent characterizations of invertibility.

Next Question

True or False #14

If A is $n \times n$ and not invertible, then $A\mathbf{x} = \mathbf{b}$ has either no solutions or infinitely many solutions.

Show Answer

Answer #14

True. Non-invertible $\Rightarrow$ never unique for all $\mathbf{b}$; a given $\mathbf{b}$ yields 0 or ∞ solutions.

Next Question

True or False #15

If A and B are invertible $n \times n$ matrices, then $(BA)^{-1} = B^{-1}A^{-1}$.

Show Answer

Answer #15

False. $(BA)^{-1} = A^{-1}B^{-1}$ — order reverses.

Next Question

True or False #16

For any $n \times m$ matrix A, $\dim \ker(A) + \dim \operatorname{image}(A) = m$.

Show Answer

Answer #16

True. Rank–Nullity: nullity + rank = # of columns.

Go to Jeopardy Board

Jeopardy! (Chs 1–4)

Category	**100**	**200**	**300**	**400**
Linear Systems & RREF	100	200	300	400
Linear Transformations	100	200	300	400
Subspaces & Dimension	100	200	300	400
Change of Basis & Similarity	100	200	300	400

Click a dollar value to view the clue. Use the buttons on each slide to navigate.

List any two of the properties that characterize a matrix in *reduced row–echelon form (rref)*.

Show Answer   Back to Board

Linear Systems & RREF for 100 — Answer

Every nonzero row has a leading 1 (pivot); each pivot is the only nonzero in its column; pivots step right as you go down; zero rows at bottom.

Back to Board

State two ways a consistent system can look in terms of variables/pivots.

Show Answer   Back to Board

Linear Systems & RREF for 200 — Answer

Exactly one solution: all variables leading. Infinitely many: at least one free variable (non-pivot column).

Back to Board

Let A be $n \times m$. Give two rank conditions and their implications for solutions.

Show Answer   Back to Board

If $\mathrm{rank}(A) = n$, the system is consistent. If $\mathrm{rank}(A) = m$, the system has at most one solution.

Back to Board

For an $n \times n$ coefficient matrix A, state a condition on A that guarantees a unique solution for to the equation $A\vec{x} = \vec{b}$ for *any* choice of $\vec{b}$.

Show Answer   Back to Board

Linear Systems & RREF for 400 — Answer

A invertible $\Leftrightarrow \mathrm{rref}(A) = I_n \Leftrightarrow \mathrm{rank}(A) = n$; thus $A\mathbf{x} = \mathbf{b}$ has a unique solution for all $\mathbf{b}$.

Back to Board

Linear Transformations for 100

Give the two algebraic rules that characterize linearity for $T : \mathbb{R}^m \to \mathbb{R}^n$.

Show Answer   Back to Board

Linear Transformations for 100 — Answer

$T(\mathbf{v}+\mathbf{w}) = T(\mathbf{v}) + T(\mathbf{w})$ and $T(k\mathbf{v}) = k\,T(\mathbf{v})$.

Back to Board

Linear Transformations for 200

Let $T : \mathbb{R}^m \to \mathbb{R}^n$ have matrix A, and $S : \mathbb{R}^n \to \mathbb{R}^p$ have matrix B (all w.r.t. standard bases).

(i) What is the matrix of $S \circ T$? (ii) What sizes must A and B have?

Show Answer Back to Board

Linear Transformations for 200 — Answer

(i) $[S \circ T] = BA$. (ii) $A \in \mathbb{R}^{n \times m}$, $B \in \mathbb{R}^{p \times n}$, so $BA \in \mathbb{R}^{p \times m}$.

Back to Board

Linear Transformations for 300

How do you build the matrix of $T : \mathbb{R}^m \to \mathbb{R}^n$ from $T(\mathbf{e}_i)$?

Show Answer   Back to Board

Linear Transformations for 300 — Answer

Use columns: $A = \begin{bmatrix} T(\mathbf{e}_1) & T(\mathbf{e}_2) & \cdots & T(\mathbf{e}_m) \end{bmatrix}$.

Back to Board

Linear Transformations for 400

Give three equivalent criteria for an $n \times n$ matrix A to be invertible.

Show Answer

Back to Board

Linear Transformations for 400 — Answer

A invertible $\Leftrightarrow \operatorname{rref}(A) = I_n \Leftrightarrow \operatorname{rank}(A) = n \Leftrightarrow \ker A = \left\{\vec{0}\right\}.$

Back to Board

Let T be a linear transformation. State why $\mathrm{image}(T)$ and $\ker(T)$ are subspaces.

Show Answer   Back to Board

Subspaces & Dimension for 100 — Answer

Each contains $\mathbf{0}$ and is closed under addition and scalar multiplication.

Back to Board

Subspaces & Dimension for 200

Give the definition of a basis of a subspace and the meaning of *dimension*.

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Subspaces & Dimension for 200 — Answer

A basis is a spanning set with no redundancies (linearly independent). The dimension is the number of vectors in any basis.

Back to Board

Subspaces & Dimension for 300

State the Rank–Nullity Theorem for an $n \times m$ matrix A.

Show Answer   Back to Board

Subspaces & Dimension for 300 — Answer

$\dim \ker(A) + \dim \operatorname{image}(A) = m$ (nullity + rank = # columns).

Back to Board

In an m-dimensional subspace $V \subseteq \mathbb{R}^n$, when do m vectors form a basis?

Show Answer Back to Board

Subspaces & Dimension for 400 — Answer

Either (i) they are linearly independent, or (ii) they span V; in either case, they form a basis.

Back to Board

Change of Basis & Similarity for 100

What are the B-coordinates of $\mathbf{x}$ and the B-coordinate vector?

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Change of Basis & Similarity for 100 — Answer

If $B = \{\mathbf{v}_1, \ldots, \mathbf{v}_m\}$ and $\mathbf{x} = \sum c_i \mathbf{v}_i$, then $(c_1, \ldots, c_m)$ are the B-coordinates and $[\mathbf{x}]_B = (c_1, \ldots, c_m)^\top$.

Back to Board

Change of Basis & Similarity for 200

Define the B-matrix of a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^n$.

Show Answer   Back to Board

Change of Basis & Similarity for 200 — Answer

The unique matrix A with $[T(\mathbf{x})]_B = A[\mathbf{x}]_B$ for all $\mathbf{x}$.

Back to Board

What is the change-of-basis matrix $S_{B \leftarrow C}$ between two bases B and C of the same subspace V?

Show Answer   Back to Board

The (unique) matrix S satisfying $[\mathbf{v}]_B = S[\mathbf{v}]_C$ for all $\mathbf{v} \in V$.

Back to Board

Relate the B-matrix A and the B'-matrix B of the same linear map via the change-of-basis matrix S.

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Change of Basis & Similarity for 400 — Answer

They are similar: $A = S\,B\,S^{-1}$ (equivalently, $AS = SB$).

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